

Chapter 15: Section 3: Imaginary Numbers and Negative Square Roots

## Warm Up

1) Are the rational numbers closed under (add, subt, mult, div.)? If not give a counter example.
2) Write each of the following as a fraction.
a) . $345345345 . .$.
b) . 45454545 ...
c) . $\overline{3}$
3) Simplify each of the following
a) $\left(x^{4}\right)^{10}$
b) $x^{3} x^{15}$
c) $\sqrt{99}$

## 15. 3 Learning Targets

- Know the Properties and
of Imaginary Numbers
- Understand the Powers of $i$
- Rewrite expressions involving negative squareroots

| Set Name | Description | Examples | Closed <br> Under |
| :--- | :--- | :--- | :--- |
| Natural numbers | Counting numbers | $0,2,3, \ldots$ |  |
| Whole numbers | Natural numbers and the <br> number zero | Whole numbers and their <br> additive inverses | $-3,-2,-1,0,1,2,3, \ldots$ |
| Integers | All numbers that can be <br> written as $\frac{a}{b}$, where $a$ and <br> $b$ are integers | $\frac{1}{2},-\frac{4}{7}, 0.75,0.01,3,6.51$ | Closed |
| Rational numbers | All numbers that cannot <br> be written as $\frac{a}{b}$, where $a$ and <br> $b$ are integers | $\sqrt{2}, \pi, 0.342359 \ldots$ |  |
| Irrational numbers | All rational and irrational <br> numbers | $\frac{3}{2}, 0,4, \sqrt{5}, 3.222,-10$ |  |
| Real numbers |  |  |  |

## Exponentiation

Is the process of taking a quantity to a power.

$$
4^{2}=\quad(-4)^{2}=\quad 4^{\frac{1}{2}}=
$$

Is the set of $\mathbb{R}$ closed for the process exponentiation?

> No! Counter Example: $(-4)^{\frac{1}{2}}=$ which is not a $\mathbb{R}$.

## The Number $\boldsymbol{i}$

## The number $i$ is a number such that $i^{2}=-1$

So...
$i=$

## Why $i$ ?

So is the set of $\mathbb{R}$ closed for the process taking $\sqrt{ }$ ?

No! Counter Example: $\sqrt{-4}=$ ? ? ? which is not a $\mathbb{R}$.

## The Powers of $i$

Write the values of the first four powers of $i$.
a. $i^{1}=$
b. $i^{2}=$
c. $i^{3}=$
d. $i^{4}=$

Use your answers to calculate each power of $i$.
a. $i^{5}=$
b. $i^{6}=$
c. $i^{7}=$
d. $i^{8}=$

## The Powers of $\boldsymbol{i}$

The powers of $i$ follow a repeating pattern and can be found by determining the remainder after dividing the power by 4


## Calculate each power of $i$.

a. $i^{101}=$
b. $i^{102}=$
c. $i^{i^{\circ}}=$
d. $i^{12}=$

## Simplifying Radical Expressions

Break the inside of a radical into factors that can be square rooted. (look for pairs in prime factorization)

$$
\begin{gathered}
\sqrt{18 x^{2} y} \\
\sqrt{2 \cdot 3 \cdot 3 \cdot x \cdot y} \\
\sqrt{3^{2}} \sqrt{x^{2}} \sqrt{2 y} \\
3 \cdot x \cdot \sqrt{2 y}
\end{gathered}
$$

## Using $i$ to Simplify $\sqrt{ }$

Since $\boldsymbol{i}=\sqrt{-1}$ we can now simplify radical expressions that involve taking the square root of a negative number

$$
\begin{gathered}
\sqrt{-49} \\
\sqrt{-1 \cdot 49} \\
\sqrt{49} \cdot \sqrt{-1} \\
7 i
\end{gathered}
$$

Simplify each expression by using $i$.
a. $\sqrt{-4}=$
b. $\sqrt{-12}=$
c. $5+\sqrt{-50}=$
d. $\frac{6-\sqrt{-8}}{2}=$

## The Imaginary Number System

 The set of Imaginary numbers can be written...$$
a+b i
$$

where $b \neq \mathbf{0} \& a, b \in \mathbb{R}$
(If there is no a then we say it is purely imaginary)

## The Complex Number System

The number system that consists of all real numbers, all imaginary numbers and their overlap. They can be written...

$$
a+b i
$$

where $a, b \in \mathbb{R}$

## ( $a$ is called the real part and bi is the imaginary)



Chapter 15.4: Complex Numbers, Complex Quotient \& Conjugate

## Warm Up

1) Are the irrational numbers closed under (add, subt, mult, div.)? If not give a counter example.
2) Write each of the following as a fraction:
a) . $311311 .$.
b) . 4242 ...
c).$\overline{4}$
3) Simplify each of the following
a) $12+\sqrt{-25}$
b) $\sqrt{-99}$
4) Simplify each power of $i$ :
a) $i^{21}$
b) $i^{30}$
c) $\boldsymbol{i}^{16}$
d) $\boldsymbol{i}^{17}$

## Chapter 15 Section 4 Learning Target

## and of Complex Numbers

- Operations on imaginary and Complex numbers
- Rewriting complex quotient (complex conjugate)



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## Adding and Subtracting Complex \#'s

For complex numbers $a+b i \& c+d i$,
$(a+b i)+(c+d i)=(a+c)+(b+d) \boldsymbol{i}$
\&
$(a+b i)-(c+d i)=(a-c)+(b-d) i$

## Simplify

$$
(8+2 i)-(7-5 i)
$$

## Multiplying Complex \#'s

For complex numbers $a+b i \& c+d i$,

$$
\begin{gathered}
(a+b i)(c+d i)= \\
a c+a d i+c b i+b d\left(i^{2}\right)
\end{gathered}
$$

$$
-1
$$

$(3+2 i)(4+5 i)$

## Try it...Practice

- $(4+6 i)(4-6 i)$
- $(5+8 i)(5-8 i)$


## Complex Conjugate

Complex Conjugates are pairs of numbers that take the form $(a+b i)$ and $(a-b i)$.

$$
\begin{gathered}
(5+3 \boldsymbol{i})(5-3 i) \\
5^{2}-\mathbf{1 5 i}+\mathbf{1 5 i}-(3 \boldsymbol{i})^{2} \\
5^{2}-3^{2} \boldsymbol{i}^{2} \\
5^{2}+3^{2}
\end{gathered}
$$

The product of a pair of complex conjugates is always a real number and equal to $a^{2}+b^{2}$

## Rewriting Complex Quotient

Recall that $i=\sqrt{-1} \ldots$ this causes issues when dividing by complex numbers with an imaginary part


This can be fixed by using the complex conjugate

## Try it...Practice...

1. 
2. 
